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Research note

Plasma flow characteristics in converging field line geometry in anisotropic plasmas

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Abstract: Plasma flow characteristic in the anisotropic plasmas was examined in converging field lines by assuming the double adiabatic condition. When the pressure anisotropy disappears, the plasma flows behave as isotropic fluids; flows are decelerated/accelerated above/below the sound velocity. When the pressure anisotropy defined by $P_\perp/P_z$ was below unity, the critical velocity that separates acceleration and deceleration increased above the sound velocity. In the opposite case where $P_\perp/P_z>1$, the critical velocity decreased below the sound velocity. We suggest that a kinetic energy of the flow varies as the pressure anisotropy changes.

key words: field-aligned plasma flows, anisotropic plasmas, converging field lines

1. Introduction

It is natural to consider that plasma pressures are often anisotropic in magnetized plasmas. In this research note, we examine the field-aligned flow in the converging field geometry by assuming anisotropic plasma pressure condition. We assumed double adiabatic equations of state, where perpendicular and parallel motion with respect to the field lines is decoupled. We show a role of pressure ratio (perpendicular to parallel) on the plasma flows. Present results could be applied to the plasma flows along the field-lines, particularly in low altitudes of Earth's dipole fields.

2. Plasma flow in converging field lines

We assume anisotropic plasma pressures ($P_\perp \neq P_z$). Here, $\perp$ and z denote "perpendicular" and "parallel" to the field lines. We discuss the field-aligned behavior of the steady state plasma flow.

The equation of motion along the field lines can be written as,

$$\rho(V\cdot\nabla)V=-\nabla P_z+\frac{P_z-P_\perp}{B}\nabla B. \tag{1}$$

Here, B is the strength of the magnetic field, ρ is mass density, V is a field-aligned component of the fluid velocity, ∇ is a gradient operator along the field lines. The second term of the right-hand side of the equation describes the net parallel pressure force associated with the changing area along the flux tube and the magnetic mirror force averaged over the particles (Comfort, 1988). This term, however vanishes for the isotropic plasmas.

Plasmas considered are magnetized and motions normal and along the field lines are assumed to be decoupled. Such conditions are represented by the double adiabatic equations of state (*e.g.*, Parks, 1991)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P_\perp}{\rho B}\right)=0, \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P_z B^2}{\rho^3}\right)=0. \tag{3}$$

The equation of the mass conservation along the converging field line geometry can be expressed as

$$\rho \cdot A \cdot V=\text{const.} \tag{4}$$

Here, A denotes a cross-section of the flux tube.

Equations (1) through (4) can be written as

$$\frac{\mathrm{d}P_z}{\rho}=-V\mathrm{d}V-\frac{1}{\rho}\frac{P_\perp-P_z}{B}\mathrm{d}B, \tag{1$'$}$$

$$\frac{\mathrm{d}P_\perp}{\rho}-\frac{P_\perp}{\rho}\frac{\mathrm{d}\rho}{\rho}-\frac{P_\perp}{\rho}\frac{\mathrm{d}B}{B}=0, \tag{2$'$}$$

$$\frac{\mathrm{d}P_z}{P_z}-3\frac{\mathrm{d}\rho}{\rho}+2\frac{\mathrm{d}B}{B}=0, \tag{3$'$}$$

$$\frac{\mathrm{d}\rho}{\rho}+\frac{\mathrm{d}A}{A}+\frac{\mathrm{d}V}{V}=0. \tag{4$'$}$$

To merge eq. (1$'$) through (4$'$), we assume a conservation of magnetic flux along flux tube.

$$B \cdot A=\text{const.} \tag{5}$$

After few steps of algebraic manipulations, we have following expression that describes flow velocity in the changing area of flux tube.

$$\frac{P_\perp}{P_z}\frac{\mathrm{d}A}{A}=\frac{1}{V}\left(\frac{V^2}{C_z^2}-3\right)\mathrm{d}V. \tag{6}$$

Here, C_z is a sound velocity along the field lines defined by $\sqrt{P_z/\rho}$.

The expression of $P_\perp/P_z$ and C_z^2 in eq. (6) can be obtained from double adiabatic eqs. (2) and (3) by incorporating initial conditions at the starting point of the flow in the converging nozzle. In addition, we utilize in equations pressure anisotropy α defined by $P_\perp/P_z$ and total pressure P_t defined by $2P_\perp+P_z$, instead of using $P_\perp$, and P_z. Then, $P_\perp/P_z$ can be written as $\beta_2\ (V^2/A)$, and $1/C_z^2$ as $\beta_1 V^2$. Here, $\beta_1=(2\alpha_0+1)\ 1/(C_0^2 \cdot V_0^2)$ and $\beta_2=\alpha_0(A_0/V_0^2)$ are calculated by use of initial pressure anisotropy α_0, initial sound velocity C_0 defined by $\sqrt{P_{0t}/\rho_0}$, initial flow velocity V_0, and initial cross-section A_0.

Finally, eq. (6) may be written as,

$$\beta_2 \frac{dA}{A^2} = (\beta_1 V^4 - 3)\frac{dV}{V^3}. \tag{7}$$

By integrating eq. (7) from A_0 to A_1 of the cross section, and from V_0 to V_1 of the flow velocity, we have following expression.

$$\alpha_0 \int_1^{a_1} \frac{da}{a^2} = (2\alpha_0 + 1) M_0^2 \int_1^{v_1} v dv - 3\int_1^{v_1} \frac{dv}{v^3}.$$

Here, $a_1 = \frac{A_1}{A_0}$, $v_1 = \frac{V_1}{V_0}$, $\alpha_0 = \frac{P_{0\perp}}{P_{0z}}$ and $M_0 = \sqrt{\frac{V_0^2}{C_0^2}}$.

Finally, we have following relation.

$$a_1 = -\frac{2\alpha_0 v_1^2}{M_0^2(2\alpha_0+1)v_1^4 - [2+2\alpha_0+1+M_0^2(2\alpha_0+1)]v_1^2+3}. \tag{8}$$

Again, α_0 is pressure anisotropy defined by $\alpha_0 = P_{0\perp}/P_{0z}$, and M_0 is Mach number defined by $M_0 = \sqrt{V_0^2/C_0^2}$. Those values α_0 and M_0 are initial values where the flow started.

We analyze eq. (8) to examine whether the initial flow at $A = A_0$ is accelerated $((da_1/dv_1) < 0)$ or decelerated $((da_1/dv_1) > 0)$. It is a function of pressure anisotropy α_0 and Mach number M_0. When pressure anisotropy is unity at a flow start, the flow below/above $M_0 = 1$ is accelerated/decelerated as is shown in Fig. 1. It is a well-known result in fluid dynamics where the supersonic/subsonic flows are decelerated/accelerated in the converging nozzle (Ikui and Matsuo, 1995). When the initial pressure anisotropy α_0 decreases below unity, the flow above $M_0 = 1$ can be accelerated. Mach number that separates acceleration and deceleration of the flow (critical Mach number) varies as a function of initial pressure anisotropy α_0 as is shown in Fig. 1.

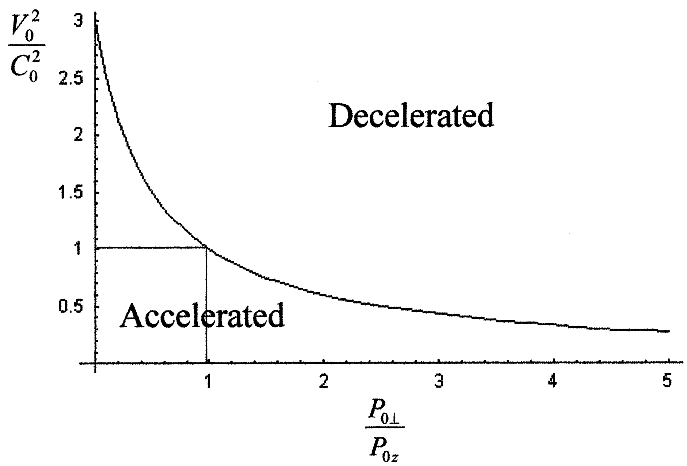

Fig. 1. A line of critical Mach number as a function of pressure anisotropy in the horizontal axis and Mach number in the vertical axis. Flows above the critical Mach number are decelerated. The vertical and horizontal lines indicate that the critical Mach number becomes unity when $P_{0\perp} = P_{0z}$.

3. Discussion

In some analogy to fluid dynamics, we could demonstrate that deceleration takes place when flow velocity exceeds the critical Mach number, M_c, because of a compressive nature in converging field line geometry. Conversely, acceleration takes place because flow is in-compressive when flow velocity is below the critical Mach number. The magnetic mirror force is force acting on a single particle. It is a multiple of a perpendicular kinetic energy and gradient of the field magnitudes. When the magnetic mirror force of individual particle is averaged over the particles in volume element, it can be written as $(-P_\perp/B)\nabla_z B$. In addition, a net parallel pressure associated with the changing area along the flux tube can be shown as $(P_z/B)\nabla_z B$ (*e.g.* Comfort, 1988). Therefore the second term of the right-hand side of the eq. (1) is consisted of the magnetic mirror force averaged over the particles and net parallel pressure force. It was found that the net parallel pressure raise the critical Mach number while the magnetic mirror force lower it. We have demonstrated the initial flow condition in the flux tube in terms of acceleration/deceleration of flow. However, the flow penetrates some distance after it leaved the starting point by changing the pressure anisotropy thereafter. The depth of the penetration will be deep when the flow is initially accelerated, while it is shallow when the flow is initially decelerated. Such a flow characteristic will be a subject of our future work.

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