Probability of occurrence of extreme magnetic storms

Ryuho Kataoka[1,2]

Received 23 February 2013; revised 23 March 2013; accepted 25 March 2013.

[1] To calculate the probability of extreme magnetic storms in the solar cycle 24, cumulative distribution functions are investigated using an 89 year list of magnetic storms recorded at Kakioka Magnetic Observatory. It is found that the probability of occurrence of extreme magnetic storms can be modeled as a function of maximum sunspot number of a solar cycle, and the probability of another Carrington storm occurring within the next decade is estimated to be 4–6%.

Citation: Kataoka, R. (2013), Probability of occurrence of extreme magnetic storms, *Space Weather*, *11*, doi:10.1002/swe.20044.

1. Introduction

[2] The Carrington event of 1859 [*Carrington*, 1859] is the largest known example of extreme space weather events that we have not experienced within the space era. Auroral activities were globally enhanced on 1–2 September 1859, including the sightings of aurora at as low as 23° magnetic latitude (Hawaii and Santiago) [*Kimball*, 1960], in association with the negative H-component excursion of dH = 1600 nT recorded at Bombay magnetogram [*Tsurutani et al.*, 2003]. The minimum *Dst* index of the Carrington storm has been estimated to be approximately −850 nT, only a half of the dH excursion [*Siscoe et al.*, 2006]. Since the beginning of the space age in 1958, the largest magnetic storm reached the minimum *Dst* index of −589 nT on 13 March 1989, which led to the collapse of the Hydro-Quebec high-voltage power transmission system in Canada [*Bolduc*, 2002]. The March 1989 storm is estimated to occur once every 60 years [*Tsubouchi and Omura*, 2007]. The probability of occurrence of such extreme events has been of great interest for the space weather community, and it was reported recently that the probability of another Carrington event occurring within the next decade could be as high as 12% [*Riley*, 2012]. The Poisson occurrence probability of *Riley* [2012] is the most likely estimate by extrapolating from smaller events, and the uncertainties associated with these types of statistical analyses are warranted by *Love* [2012].

[3] The somewhat high probability of 12% is likely to be overestimated for the current weak solar cycle 24, since the statistical analysis of *Riley* [2012] was based on a 55 year long record of *Dst* index when solar and geomagnetic activities were relatively high. The weak polar field strength observed during the last solar minimum [*Kataoka and Miyoshi*, 2010] has resulted in the weak interplanetary magnetic field in the solar cycle 24, which can be the smallest in the last 100 years [*Svalgaard et al.*, 2005]. In fact, geomagnetic activity has been unprecedentedly quiet since the beginning of the solar cycle 24, and the provisional and real-time *Dst* indices have not exceeded −150 nT so far at the time of writing of this paper (March 2013). The maximum sunspot number of solar cycle 24 is expected to be smaller than that of the previous cycle, and the possible rapid decrease in the cycle-averaged sunspot number may even be indicative that we are about to enter another grand minimum in solar activity, a period of prolonged sunspot absence, for the next several decades [*Lockwood et al.*, 2011]. The motivation of this paper is to reevaluate the probability of another Carrington storm by incorporating weak solar cycle 24, which will likely be of great interest to a broad space weather community. To estimate a more realistic probability, it is important to include geophysical data obtained during a similarly weak solar cycle as to the solar cycle 24 into our analysis.

[4] Kakioka Magnetic Observatory, Japan, is located at 27° magnetic latitude and has an 89 year record of magnetic storms since February 1924. This year 2013 is the hundredth anniversary of the Kakioka Magnetic Observatory, but all of the data before 1924 were burned out in Tokyo during the Great Kanto earthquake of 1923. The unique data set is used to extend the statistical analysis of *Riley* [2012], including the solar cycle 16 (1923–1933) which was the weakest solar cycle in the last 100 years. Kakioka Magnetic Observatory identifies two types of magnetic storms, i.e., Ssc (sudden commencement) storms and Sg (gradual commencement) storms. An Ssc storm is

[1]Interactive Research Center of Science, Tokyo Institute of Technology, Tokyo, Japan.
[2]National Institute of Information and Communications Technology, Tokyo, Japan.

Corresponding author: R. Kataoka, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro, Tokyo 152-8550, Japan. (ryuho@geo.titech.ac.jp)

1542-7390/13/10.1002/swe.20044

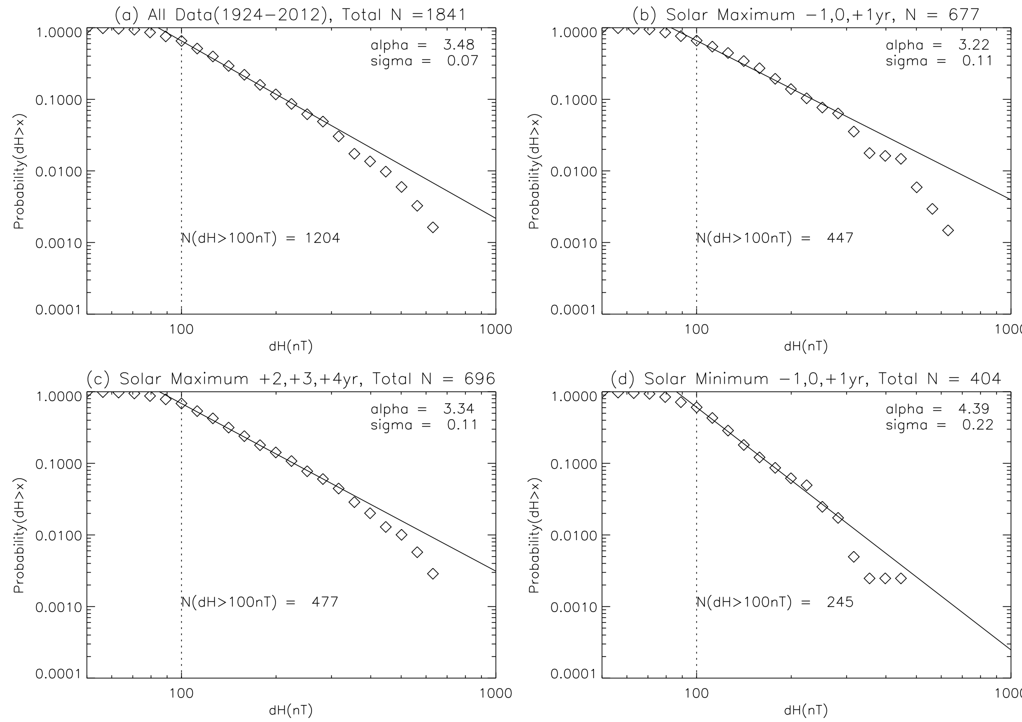

Figure 1. Cumulative distribution functions of magnetic storms as a function of the dH amplitude for the time intervals of (a) all data, (b) solar maximum, (c) early declining phase, and (d) solar minimum. The solid straight line is a fit to the data above the lower threshold $x_{\min} = 100$ nT.

identified when the K-index is greater than or equal to 5 and dH is greater than or equal to 40 nT in conjunction with a storm sudden commencement. If no sudden commencement occurs, an Sg storm is registered when the K-index is greater than or equal to 5 at least twice and dH is greater than or equal to 50 nT. Both Ssc and Sg are used in this study without discrimination because large storms are not always preceded by interplanetary shocks or sudden commencements [*Kataoka and Miyoshi*, 2006]. The magnitude of magnetic storms is ranked by the amplitude of the dH excursion. For example, dH = 644 nT on 13 March 1989, which is also the largest event after the space age, and the third largest in the list. The largest storm registered dH > 700 nT on 4 July 1941, and the second largest registered dH = 661 nT on 24 March 1940. Although the data are local, one important advantage in terms of space weather forecast is that it can be more directly compared to geomagnetically induced currents in Japan, which is roughly proportional to the amplitude of horizontal magnetic field rather than the time derivatives [*Watari et al.*, 2009; *Pulkkinen et al.*, 2010].

[5] In this paper, a new statistical model of cumulative distribution functions is constructed based on the 89 year list of magnetic storms recorded at Kakioka Magnetic Observatory. Using the dependence of a cumulative distribution function on the maximum sunspot number of a solar cycle, a more realistic probability of extreme magnetic storms is estimated.

2. Method

[6] As also shown by *Riley* [2012], a superior method of plotting the data is to calculate a cumulative distribution function. Instead of plotting a simple histogram of magnetic storms, it is useful to make a plot of probability $P(x)$ that x has a value greater than or equal to x:

$$P(x) = \int_x^\infty p(x')\,\mathrm{d}x'. \tag{1}$$

[7] If the distribution follows a power law $p(x) = Cx^{-\alpha}$, then

$$P(x) = \frac{C}{\alpha - 1} x^{-(\alpha-1)}. \tag{2}$$

[8] Thus, the cumulative distribution function $P(x)$ also follows a power law, but with a different exponent $\alpha - 1$. The normalization requirement gives the constant $C = (\alpha - 1)\, {x_{\min}}^{\alpha-1}$. A simple fitting method for extracting the exponent α is to employ the formula

$$\alpha = 1 + n\left[\sum_{i=1}^{n} \ln \frac{x_i}{x_{\min}}\right]^{-1}. \tag{3}$$

[9] Here the quantities x_i, $i = 1, \ldots, n$ are the measured values of x and $x_{\min}$ is the minimum value of x. In this study, $x_{\min}$ corresponds not to the smallest value of x measured but to the smallest for which the power-law behavior holds. An

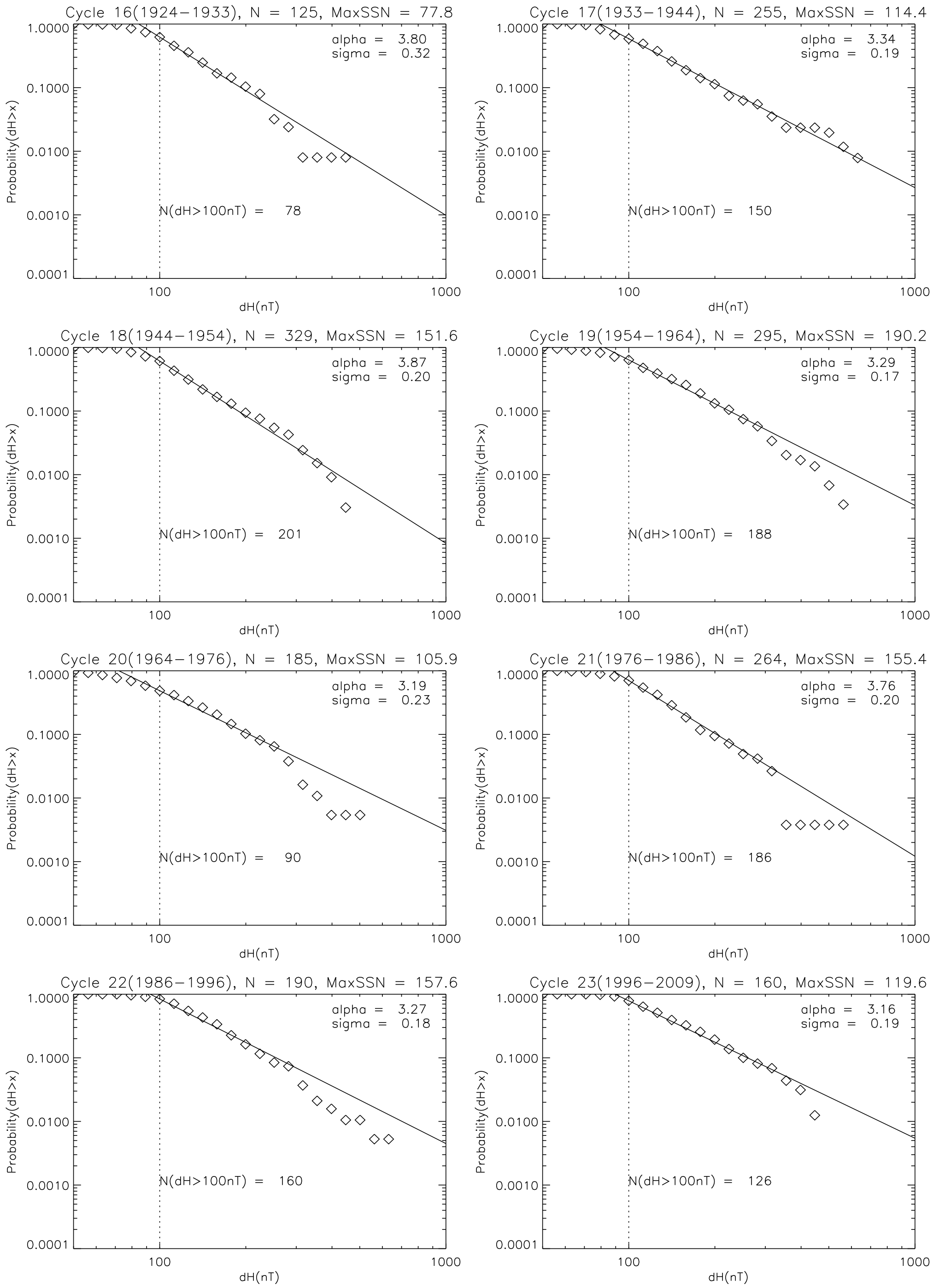

Figure 2. As Figure 1 except that the time intervals are for solar cycles 16–23. The yearly averaged maximum sunspot number is shown at the upper right of each panel.

estimate of the expected statistical error on equation (3) is given by

$$\sigma = \sqrt{n}\left[\sum_{i=1}^{n} \ln \frac{x_i}{x_{\min}}\right]^{-1} = \frac{\alpha - 1}{\sqrt{n}}. \qquad (4)$$

[10] The derivation of these formulas is given in *Newman* [2005].

[11] Assuming that the events occur independently of one another, the Poisson distribution can be used to infer the probability of the events greater than x_{crit} occurring during some time Δt:

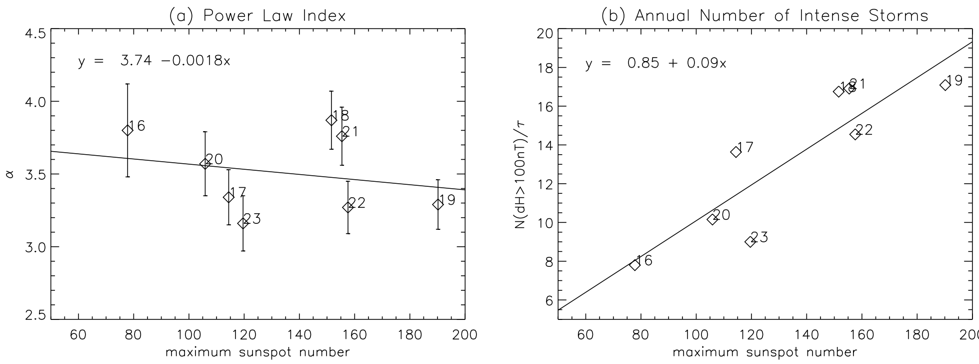

Figure 3. (a) The slope α of the power law and the error σ for solar cycles 16–23. (b) The number of intense storms divided by the cycle length for solar cycles 16–23. The straight lines show the least square fit as a function of maximum sunspot number.

$$P_C(x \geq x_{crit}, t = \Delta t) = 1 - e^{-n\frac{\Delta t}{\tau}P(x_{crit})}, \quad (5)$$

where τ is the total time interval of the data set. The set of equations (2)–(5) is a robust method for computing the probability that an event exceeding x_{crit} will occur within the next Δt years. More detailed discussion of the method of analysis was described in *Riley* [2012].

3. Results and Discussion

[12] The cumulative distribution function of magnetic storms is calculated as a function of the dH amplitude using the whole 89 year data set in Figure 1a. The solid straight line is a fit to the data above the lower threshold $x_{min} = 100$ nT using equation (3). It is found that the cumulative distribution function can be modeled as a power law. The total number of intense storms (dH > 100 nT) is 1204, and the fitted slope is $\alpha = 3.48$. Using these parameters and equation (5), the probability of another Carrington storm (dH = 1600 nT) within the next decade ($\Delta t = 10$ years) is estimated to be 13%, which is consistent with the results of *Riley* [2012] who also discussed the consistency with other space weather events (flare intensity, coronal mass ejection speeds, *Dst*, proton events as inferred from nitrate records). Since the actual cumulative distribution function is always lower than the fitted power law at super storms of dH > 300 nT, the estimated probability of 13% gives the upper limit. Although time stationary has been assumed, another problematic limitation is that the distribution largely changes within a solar cycle, depending on the phase of a solar cycle as shown in Figures 1b–1d. The probabilities of another Carrington storm within the next decade are estimated to be 32.6%, 24.7%, and 0.8% with regard to Figures 1b, 1c, and 1d, respectively. It is therefore found that the probability of super storm occurrence is overestimated for solar minimum and is underestimated for solar maximum and declining phase. It is also found from Figure 1d that the significant deviation from the power law at dH > 300 nT in Figure 1a is mainly caused by the lack of such events during the solar minimum. The gradual deviation from the power law at dH > 300 nT in Figures 1c and 1d may reflect various kinds of complicated saturation processes working in the magnetosphere during super storms as discussed by *Kataoka et al.* [2005, and references therein].

[13] In Figure 2, the cumulative distribution functions are calculated for each solar cycle to see the cycle to cycle variation. It is found that the cumulative distribution function can be well modeled as a power law in every solar cycle, and the distribution deviates when there are only a few events. It is also found that the number of intense storms (dH > 100 nT) is roughly proportional to the maximum sunspot number of a solar cycle, while the slope of the power law does not dramatically change over the eight solar cycles.

[14] The cycle to cycle variations can be summarized as follows. In Figure 3, it is found that the slope α shows a weak dependence on the maximum sunspot number, while the number of intense storms (dH > 100 nT) divided by the cycle length is more clearly proportional to the maximum sunspot number of a solar cycle. Using the results of the least square fit, we formulate the probability of super storms, i.e., equation (5), as a function of the slope α and the number of intense storms n (dH > 100 nT) divided by the cycle length τ, where both parameters are described to be linearly proportional to the maximum sunspot number N_S as follows:

$$\frac{n(\mathrm{dH} > 100\ \mathrm{nT})}{\tau} = 0.85 + 0.09N_S, \quad (7)$$

$$P(x_{crit}) = \left(\frac{x_{crit}}{100\ \mathrm{nT}}\right)^{-(2.74-0.0018N_S)}. \quad (8)$$

[15] Although the actual maximum sunspot number will be determined in a few years, we need to select the maximum sunspot number for solar cycle 24 from some predicted values to apply this statistical model. The maximum sunspot number of the solar cycle 24 was predicted to be as low as 75 by *Svalgaard et al.* [2005], while based on a precursor method, it was predicted to be 84 [*Yoshida and Yamagishi*, 2010]. Although many other predictions are reviewed by *Pesnell* [2012], it is noteworthy that the prediction method by *Yoshida and Yamagishi* [2010] is one of the simplest, and it is consistent with many important

observations of the weak polar field [*Svalgaard et al*., 2005] as well as the cycle length [*Hathaway et al*., 2002; *Watari*, 2008]. It is also consistent with the longer-term observation of cycle length as reconstructed from tree rings, i.e., a roughly 14 year cycle length was found in ^{14}C content of tree rings formed during the Maunder Minimum when the sunspots mostly disappeared, while a 9 year length was found during the Early Medieval Maximum Period (ninth to tenth century) when the solar activity was estimated to be persistently higher than average [*Miyahara et al*., 2004, 2007, 2008]. Taking the expected maximum sunspot number of 84 for the solar cycle 24, and using equations (5), (7), and (8), the probability of another Carrington storm (dH = 1600 nT) occurring within the next decade ($\Delta t = 10$ years) is estimated to be 6%. Even if the solar maximum had already passed, and using the yearly averaged sunspot number 58 of 2012 as the maximum sunspot number for solar cycle 24, the probability of another Carrington storm is estimated to be 4%. It is therefore found that the probability of extreme magnetic storms is still not negligibly small even for the current weak solar cycle 24, although the probability is predicted to be less than a half of the 12% previously estimated by *Riley* [2012].

[16] Torahiko Terada (1878–1935), who carefully selected the location of Kakioka Magnetic Observatory 100 years ago, has been a popular physicist in Japan by leaving words like "We are unprepared because natural disasters are simply very rare, so just when we have forgotten one mistake we get ready to make another." Two years have passed since the 3.11 earthquake of 2011, the words sound meaningfully again. We recently experienced a surprising event on 15 February 2013 when more than 1000 people have been injured by shock waves of a small asteroid over Chelyabinsk, by chance at a close timing of the 45 m asteroid 2012DA14 flyby. It has been estimated that objects as large as 50 m (comparable with the asteroid causing the Tunguska event of 1908) hit the Earth roughly once every thousand years [*Chapman and Morrison*, 1994], and the probability of such an impact over the next decade can be estimated to be 1% [*Riley*, 2012], which is slightly smaller than the revised probability of another Carrington event in this paper. It would be worthwhile to compare these two scenarios to consider the policy against another Carrington event. Although the occurrence rates are now clear for both scenarios, possible damages of another Carrington event are relatively unclear due to the complicated mechanisms. As a next step, we are trying to obtain realistic space weather simulations to understand the complicated physical mechanism of extreme events, which can contribute to shift the present high-tech society to more robust directions.

[17] **Acknowledgments.** The author would like to thank S. Watari, S. Fujita, Y. Minamoto, and Y. Ebihara for valuable discussions, NOAA/NGDC for the sunspot numbers, and Kakioka Magnetic Observatory for providing the list of magnetic storms.

References

Bolduc, L. (2002), GIC observations and studies in the Hydro-Quebec power system, *J. Atmos. Sol.-Terr. Phys.*, *64*, 1793–1802.

Carrington, R. C. (1859), Description of a singular appearance seen in the Sun on September 1, 1859, *Monthly Notices of the Royal Astronomical Society*, *20*, 13–15.

Chapman, C. R., and D. Morrison (1994), Impacts on the Earth by asteroids and comets: Assessing the hazard, *Nature*, *367*, 33–40.

Hathaway, D. H., R. M. Wilson, and E. J. Reichmann (2002), Group sunspot numbers: Sunspot cycle characteristics, *Solar Phys.*, *211*, 357–370.

Kataoka, R., D. H. Fairfield, D. G. Sibeck, L. Rastatter, M.-C. Fok, T. Nagatsuma, and Y. Ebihara (2005), Magnetosheath variations during the storm main phase on 20 November 2003: Evidence for solar wind density control of energy transfer to the magnetosphere, *Geophys. Res. Lett.*, *32*, L21108, doi:10.1029/2005GL024495.

Kataoka, R., and Y. Miyoshi (2006), Flux enhancement of radiation belt electrons during geomagnetic storms driven by coronal mass ejections and corotating interaction regions, *Space Weather*, *4*, S09004, doi:10.1029/2005SW000211.

Kataoka, R., and Y. Miyoshi (2010), Why are relativistic electrons persistently quiet at geosynchronous orbit in 2009?, *Space Weather*, *8*, S08002, doi:10.1029/2010SW000571.

Kimball, D. S. (1960), A study of the aurora of 1859, University of Alaska, UAG-R109.

Lockwood, M., M. J. Owens, L. Barnard, C. J. Davis, and F. Steinhilber (2011), The persistence of solar activity indicators and the descent of the Sun into Maunder Minimum conditions, *Geophys. Res. Lett.*, *38*, L22105, doi:10.1029/2011GL049811.

Love, J. F. (2012), Credible occurrence probabilities for extreme geophysical events: Earthquakes, volcanic eruptions, magnetic storms, *Geophys. Res. Lett.*, *39*, L10301, doi:10.1029/2012GL051431.

Miyahara, H., K. Masuda, Y. Muraki, K. H. Furuzawa, H. Menjo, and T. Nakamura (2004), Cyclicity of solar activity during the Maunder Minimum deduced from radiocarbon content, *Solar Phys.*, *224*(1–2), 317–322, doi:10.1007/s11207-005-6501-5.

Miyahara, H., K. Masuda, K. Nagaya, K. Kuwana, Y. Muraki, and T. Nakamura (2007), Variation of solar activity from the Spoerer to the Maunder minima indicated by radiocarbon content in tree rings, *Adv. Space Res.*, *40*(7), 1060–1063, doi:10.1016/j.asr.2006.12.044.

Miyahara, H., Y. Yokoyama, and K. Masuda (2008), Possible link between multi-decadal climate cycles and periodic reversals of solar magnetic field polarity, *Earth and Planetary Science Letters*, *272*(1–2), 290–295.

Newman, M. E. J. (2005), Power laws, Pareto distributions and Zipf's law, *Contemporary Physics*, *46*, 323–351.

Pesnell, W. D. (2012), Solar cycle predictions (invited review), *Solar Phys.*, *281*, 507–532, doi:10.1007/s11207-012-9997-5.

Pulkkinen, A., R. Kataoka, S. Watari, and M. Ichiki (2010), Modeling geomagnetically induced currents in Hokkaido, Japan, *Adv. Space Res.*, *46*(9), 1087–1083, doi:10.1016/j.asr.2010.05.024.

Riley, P. (2012), On the probability of occurrence of extreme space weather events, *Space Weather*, *10*, S02012, doi:10.1029/2011SW000734.

Siscoe, G., N. U. Crooker, and C. R. Clauer (2006), *Dst* of the Carrington storm of 1859, *Adv. Space Res.*, *38*, 173–179, doi:10.1016/j.asr.2005.02.102.

Svalgaard, L., E. W. Cliver, and Y. Kamide (2005), Sunspot cycle 24: Smallest cycle in 100 years?, *Geophys. Res. Lett.*, *32*, L01104, doi:10.1029/2004GL021664.

Tsubouchi, K., and Y. Omura (2007), Long-term occurrence probabilities of intense geomagnetic storm events, *Space Weather*, *5*, S12003, doi:10.1029/2007SW000329.

Tsurutani, B. T., W. D. Gonzalez, G. S. Lakhina, and S. Alex (2003), The extreme magnetic storm of 1–2 September 1859, *J. Geophys. Res.*, *108*(A7), 1268, doi:10.1029/2002JA009504.

Watari, S. (2008), Forecasting solar cycle 24 using the relationship between length and maximum sunspot number, *Space Weather*, *6*, S12003, doi:10.1029/2008SW000397.

Watari, S., M. Kunitake, K. Kitamura, T. Hori, T. Kikuchi, K. Shiokawa, N. Nishitani, R. Kataoka, Y. Kamide, T. Aso, Y. Watanabe, and Y. Tsuneta (2009), Measurement of geomagnetically induced current in a power grid in Hokkaido, *Japan, Space Weather*, *7*, S03002, doi:10.1029/2008SW000417.

Yoshida, A., and H. Yamagishi (2010), Predicting amplitude of solar cycle 24 based on a new precursor method, *Ann. Geophys.*, *28*, 417–425.