Journal of

GEOPHYSICAL RESEARCH

The continuation of

Terrestrial Magnetism and Atmospheric Electricity

VOLUME 62 DECEMBER, 1957 No. 4

SWEET'S MECHANISM FOR MERGING MAGNETIC FIELDS IN CONDUCTING FLUIDS*

BY E. N. PARKER

Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago 37, Illinois

(Received June 4, 1957)

ABSTRACT

Sweet's mechanism for the merging of two oppositely directed magnetic fields in a highly conducting fluid is investigated in a semi-quantitative manner. It is shown that two oppositely directed sunspot fields with scales of 10^4 km could be merged by Sweet's mechanism, if shoved firmly together, in about two weeks; their normal interdiffusion time would be of the order of 600 years. It is suggested that Sweet's mechanism may be of considerable astrophysical importance: It gives a means of altering quickly the configuration of magnetic fields in ionized gases, allowing a stable field to go over into an unstable configuration, subsequently converting much of the magnetic energy into kinetic energy of the fluid.

I. INTRODUCTION

Sweet (1956) has recently pointed out that when two oppositely directed magnetic fields of scale L in a highly conducting medium are shoved against each other, an interesting situation arises in which the two fields will interdiffuse in times small compared to the usual diffusion time $L^2\sigma/c^2$; σ is the conductivity in esu. Sweet's mechanism may be of importance in rapidly altering the connectivity of magnetic fields associated with activity in the solar atmosphere, etc.: For instance, a sunspot field, $L \cong 10^9$ cm, has an ordinary diffusion or decay time of

*Assisted in part by the Office of Scientific Research and the Geophysics Research Directorate, Air Force Cambridge Research Center, Air Research and Development Command, U.S. Air Force.

the order of 2×10^{10} sec, where the temperature is 10^4 degrees Kelvin and $\sigma \cong 1.8 \times 10^{13}$ esu; with Sweet's mechanism, it is conceivable that two such oppositely directed fields can be interdiffused and their lines of force reconnected in 10^4 seconds or less.

The rapid interdiffusion of two oppositely directed fields when they are pressed together by external forces arises from the fact that the field vanishes on the surface between the two oppositely directed regions, and the entire compressive stress falls on the conducting fluid. The fluid responds to the excess pressure by flowing out of the region along the lines of force, and the two oppositely directed magnetic fields approach each other more and more closely, according to the usual hydromagnetic equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \qquad (1)$$

in a medium of large electrical conductivity. Consequently, the gradient in the field density across the neutral surface between the fields increases without limit, until no matter how large may be the electrical conductivity σ, the diffusion term $(c^2/4\pi\mu\sigma)\ \nabla^2\mathbf{B}$, omitted in (1), becomes comparable to the dynamical term $\nabla \times (\mathbf{v} \times \mathbf{B})$, and the two oppositely directed fields interdiffuse as rapidly as the efflux of fluid from between the fields allows them to approach each other. The process is shown schematically in Figure 1 for two bipolar sunspot fields at the same solar latitude. Initially, the fields are widely separated and have no interconnecting lines of force. In Figure 1(a), we imagine that suitable fluid motions in the dense gases beneath the photosphere have shoved the two fields together, with the distortion shown. The high electrical conductivity of the solar atmosphere prevents interconnection of the lines of force. However, with the outflow of the gas caught between the two fields, as indicated in Figure 1(b), the gradient in $\mathbf{B}$ across the neutral plane increases until rapid interdiffusion takes place and the lines of force reconnect, as shown in Figure 1(c).

If l is the characteristic length of the gradient in $\mathbf{B}$ across the neutral plane, then the decay time of the field in the region of this gradient is of the order of $l^2\sigma/c^2$. The velocity u with which the fields merge is $l/(l^2\sigma/c^2)$,

$$u \cong c^2/l\sigma$$

The fluid expelled along the lines of force over a front of width L achieves a velocity v, where

$$v \cong uL/l$$

based on geometrical considerations. The pressure $B^2/8\pi$ available for squeezing the fluid out along the lines of force leads to the conclusion that

$$\tfrac{1}{2}\rho v^2 \cong B^2/8\pi$$

from energy considerations; hence $v \cong C_0$, where C_0 is the characteristic hydromagnetic velocity $B/(4\pi\rho)^{1/2}$. Therefore, it follows that

$$u \cong c(C_0/L\sigma)^{1/2}$$

$$l/L \cong c/(C_0L\sigma)^{1/2}$$

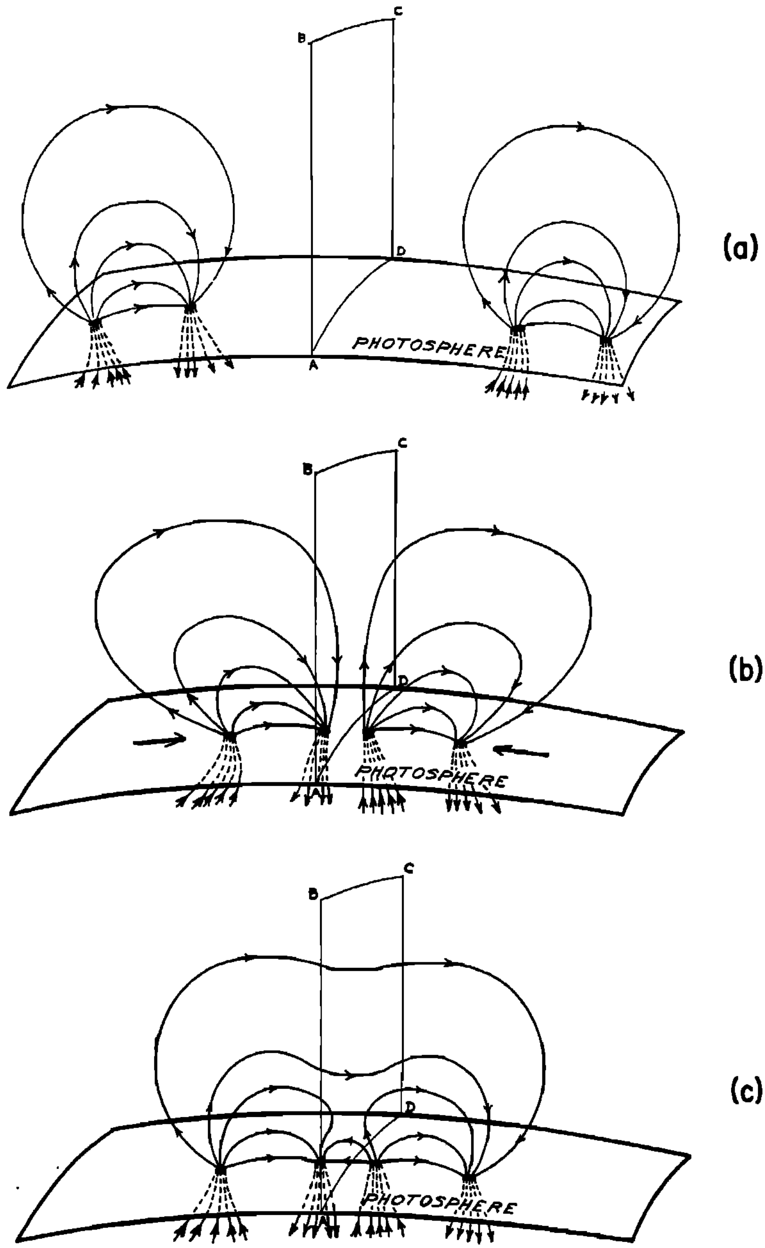

FIG. 1—(*a*) Two widely separated bipolar sunspot groups at the same solar latitudes
(*b*) The distortion of the bipolar fields as the groups are shoved together
(*c*) The reconnection of the lines of force in a week or so, as a consequence of Sweet's mechanism

Without Sweet's mechanism, the diffusion velocity would be $c^2/L\sigma$, which is equal to $(l/L)u$. For the case of two bipolar sunspot fields of 1,000 gauss, $L \cong 10^9$cm, $\sigma \cong 1.8 \times 10^{13}$ esu, and $\rho = 10^{-8}$gm/cm^3, we have $C_0 \cong 100$ km/sec, $u \cong 7$ m/sec, and $l/L \cong 0.7 \times 10^{-4}$. We see how thin, $10^{-4}L$, is the transition layer l in which the diffusion takes place.

II. EXPULSION OF FLUID

To understand the physical process involved in Sweet's mechanism, consider first how the fluid caught between two magnetic fields might be squeezed from between by pressing together the fields. Let us suppose that initially we have two infinitely conducting sheets at $x = \pm\,\epsilon$, as shown in Figure 2. We fill the thin layer between the two sheets with infinitely conducting inviscid fluid. Outside

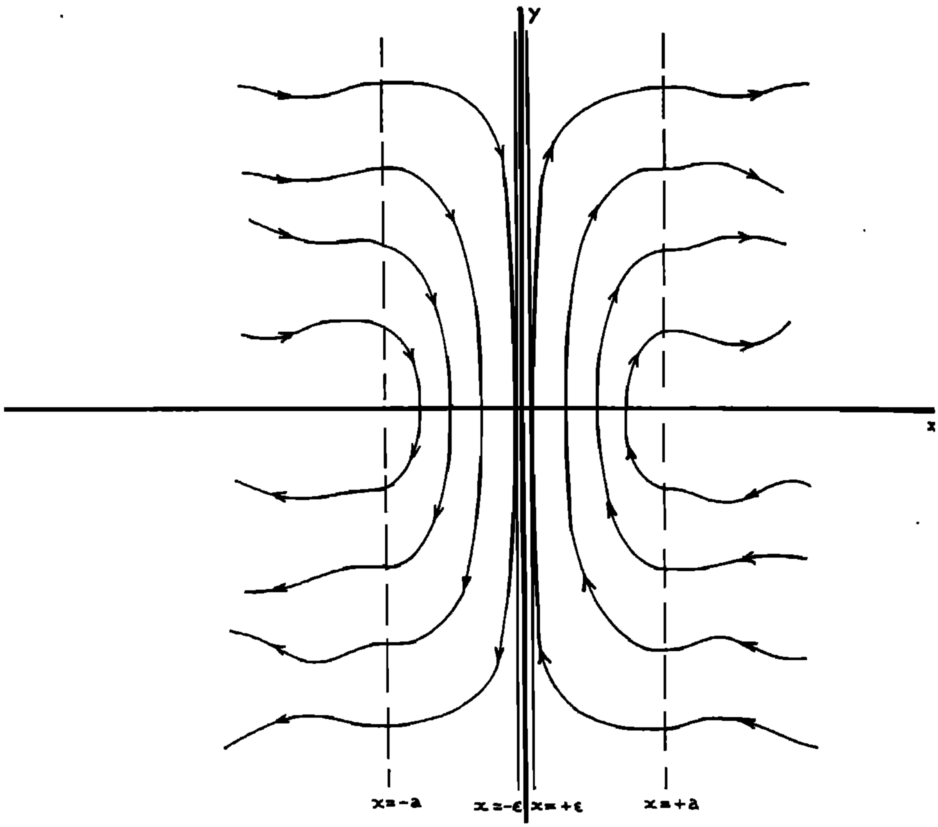

FIG. 2—Schematic diagram of two oppositely directed magnetic fields shoved against two superconducting sheets at $x = \pm\,\epsilon$ by suitable motions, beyond $x = \pm\,a$, of the infinitely conducting fluid outside the sheets at $x = \pm\,\epsilon$

the sheets, we introduce the magnetic field **B** with lines of force everywhere parallel to the xy-plane and expressible as the gradient of a scalar potential ψ,

$$\mathbf{B} = -\nabla\psi \qquad (2)$$

so that **B** exerts no force on the conducting fluid there. To fix ideas, we suppose that the field **B** is in the grip of hydrodynamic forces in the conducting fluid beyond $x = \pm\,a$ and is held in such a way that $B_y = 0$ on $x = \pm\,a$ and

$$B_x = +B_0(y/b)\exp(-y^2/b^2)$$

at $x = +a$,

$$B_x = -B_0(y/b)\exp(-y^2/b^2)$$

at $x = -a$. As a consequence of the infinitely conducting sheets at $x = \pm\epsilon$ ($\epsilon \ll a$), the field will not penetrate into $-\epsilon < x < +\epsilon$, and $B_x = 0$ on $x = \pm\epsilon$. With $\mathbf{B} = -\nabla\psi$ in $-a < x < -\epsilon$ and $+\epsilon < x < +a$, it is readily shown that

$$\psi(x, y) = \pm\frac{B_0 b^2}{4\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{dk\sin ky}{\sinh ka}\cosh k(x \pm \epsilon)\exp(-k^2b^2/4) \quad \cdots\cdots(3)$$

where $\pm$ is $+$ for $-a < x < -\epsilon$ and $-$ for $+\epsilon < x < +a$. It follows that the field density at $x = \pm\epsilon$ is

$$B_y(\pm\epsilon, y) = \mp\frac{B_0 b^2}{4\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{dk\, k\cos ky}{\sinh ka}\exp(-k^2b^2/4)$$

If the forces beyond $x = \pm a$ press the two fields on either side of $x = \pm\epsilon$ together sufficiently firmly that the fields are considerably compressed in the x-direction and $a \ll b$, then we may expand $\sinh ka$ about $k = 0$ and carry out the indicated integration, obtaining

$$\left.\begin{aligned} &B_y(\pm\epsilon, y) \sim (B_0 b/2a^2)\exp(-y^2/b^2) \\ &\times\{1 - \tfrac{1}{3}(a^2/b^2)(1 - 2y^2/b^2) + O^4(a/b)\}\end{aligned}\right\} \quad \cdots\cdots(4)$$

The pressure exerted by the magnetic field on the two superconducting sheets at $x = \pm\epsilon$ is just $p = B_y^2(\pm\epsilon, y)/8\pi$. If we remove the superconducting sheets, so that the pressure is brought to bear directly on the field-free infinitely conducting fluid in $-\epsilon < x < +\epsilon$, then the motion of an element of fluid with position $Y(t)$ is given by

$$\frac{d^2Y}{dt^2} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

Multiplying by dY/dt and integrating, we have

$$\frac{1}{2}\left(\frac{dY}{dt}\right)^2 = -\int_{Y(0)}^{Y(t)} dy\,\frac{1}{\rho}\frac{\partial p}{\partial y}$$

For incompressible flow, ρ is a constant, and

$$\frac{dY(t)}{dt} = \frac{C_0 b}{2a}\left\{\exp\left[-\frac{2Y^2(0)}{b^2}\right] - \exp\left[-\frac{2Y^2(t)}{b^2}\right]\right\}^{1/2}\left\{1 + O^2\left(\frac{a}{b}\right)\right\}$$

where C_0 is the characteristic hydromagnetic velocity $B_0/(4\pi\rho)^{1/2}$.

To compute $Y(t)$ as a function of t and $Y(0)$, we expand $Y(t)$ in ascending powers of t. We may then carry out the integration, obtaining

$$\left.\begin{aligned} Y(t) = Y(0)\Big\{1 &+ \left(\frac{C_0 t}{a}\right)^2\exp\left[-\frac{2Y^2(0)}{b^2}\right] \\ &+ \frac{1}{96}\left(\frac{C_0 t}{a}\right)^4\left[1 - \frac{4Y^2(0)}{b^2}\right]\exp\left[-\frac{4Y^2(0)}{b^2}\right] + O^6\left(\frac{C_0 t}{a}\right)\Big\}\end{aligned}\right\} \quad \cdots\cdots(5)$$

The coefficient of the term $O^4(C_0t/a)$ is small because of the numerical factor 1/96; it vanishes when $Y(0) = \frac{1}{2}b$, and is negligible for $Y(0) > \frac{1}{2}b$ because of the exponential. Therefore, we may to good approximation neglect the term $O^4(C_0t/a)$, considering only the first two terms in the expansion. It follows that

$$\left.\begin{aligned} dY(t)/dt &\cong 2C_0[Y(0)/a](C_0t/a) \exp[-2Y^2(0)/b^2] \\ &= 2[Y(t) - Y(0)]/t \end{aligned}\right\} \qquad (6)$$

$Y(0)$ may be expressed in terms of $Y(t)$, so that

$$Y(0) = Y(t)\left\{1 - \left(\frac{C_0t}{a}\right)^2 \exp\left[-\frac{2Y^2(0)}{b^2}\right] + O^4\left(\frac{C_0t}{a}\right)\right\} \qquad (7)$$

Then, writing $v_y = dY(t)/dt$, $y = Y(t)$, we have

$$v_y = 2C_0(y/a)(C_0t/a) \exp(-2y^2/b^2) + O^3(C_0t/a) \qquad (8)$$

We let $l(y, t)$ represent the width, in the x-direction, of the layer of field-free fluid at the position y and time t; we see that $l(y, 0) = 2\epsilon$. By integrating the equation of continuity $\nabla \cdot \mathbf{v} = 0$ for the fluid velocity from $x = -\frac{1}{2}l(y, t)$ to $x = +\frac{1}{2}l(y, t)$, we find that

$$\partial l(y, t)/\partial t + l(y, t)(\partial v_y/\partial y)_t = 0 \qquad (9)$$

Hence, we find from equation (8) that

$$l(y, t) = l(y, 0)\left\{1 - \left(\frac{C_0t}{a}\right)^2\left(1 - 4\frac{y^2}{b^2}\right) \exp\left(-\frac{2y^2}{b^2}\right) + O^4(C_0t/a)\right\} \qquad (10)$$

Near the x-axis, $y^2 \ll b^2$, and $l(y, t)/l(y, 0)$ is essentially independent of y; the field-free layer remains uniform as it decreases its thickness. For larger values of y^2/b^2, the decrease in thickness is not as rapid. No decrease occurs at $y^2 = b^2$, and for $y^2 > b^2$ the thickness increases; the fluid escaping from the high pressures in $y^2 < b^2$ inflates the low pressure region of $y^2 > b^2$, though, of course, the exponential factor $\exp(-2y^2/b^2)$ indicates that the inflation is not large.

III. MERGING OF FIELDS

Consider the rate at which two oppositely directed magnetic fields can merge, as a result of the squeezing out of the conducting fluid initially caught between them. Unfortunately, we are unable to solve simultaneously both the hydrodynamic equation for the motion of the fluid and the hydromagnetic equation for the magnetic field **B**. Since we have already discussed the hydrodynamic motions in a qualitative manner, we shall solve the equation for **B** in a formal manner, assuming an idealized form for the fluid motion based on the qualitative picture obtained in Section II. Thus, our final results will not be quantitative; we hope that they will represent a qualitative picture of the interdiffusion of **B**.

We shall restrict ourselves to steady-state conditions, so that $\partial/\partial t = 0$, and to the case considered in Section II, where the fields are pressed so closely together that $a \ll b$. Then, except in the region where $y^2 > b^2$, we have that v_x/v_y, B_x/B_y, and $(\partial/\partial y)/(\partial/\partial x)$ are all small, $O(a/b)$. The x-component of the hydrodynamic equation,

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{v} = -(1/\rho)\nabla[p + B^2/8\pi] + (1/4\pi\rho)(\mathbf{B}\cdot\nabla)\mathbf{B} \quad \text{.....(11)}$$

becomes

$$(\partial/\partial x)(p + B^2/8\pi) = O^2(a/b) \quad \text{.....(12)}$$

In Section I, we pointed out how thin is the transition layer, of thickness l, in which the diffusion takes place between the two oppositely directed fields. For $x^2 > l^2$, the field varies so slowly ($\partial/\partial x = O(1/a)$, as compared to $O(1/l)$, that for out present purposes we may regard $|\mathbf{B}(x, y)|$ as independent of x. Thus, we let

$$B(x, y) = B_y(\pm\epsilon, y)$$

for $x^2 > l^2$, where $B(x, y) = |\mathbf{B}(x, y)|$ and where $B_y(\pm\epsilon, y)$ is the field given in (4). We suppose that the hydrostatic pressure is p_0 at infinity. Therefore, upon integrating (12), we obtain

$$p(x, y) + B^2(x, y)/8\pi \cong p_0 + B_y^2(\epsilon, y)/8\pi \quad \text{.....(13)}$$

We cannot integrate the y-component of (11) because of the complications introduced by the non-linear term $v_x\partial v_y/\partial x$. However, in Section II, we found that the expulsion of fluid from between the two fields proceeds in an orderly and nearly uniform manner. In the region $y^2 < b^2$, we may expect to retain the essential features of the expulsion if we introduce the qualitative argument that the elongation $\partial v_y/\partial y$ of a fluid element at a given point is proportional to the velocity which the excess pressure, $p - p_0$, at the point is capable of producing. We write

$$\frac{\partial v_y}{\partial y} = \left(\frac{p - p_0}{\rho}\right)^{1/2}\frac{1}{L}$$

L is a constant of proportionality, and is $O(b)$. Using (13), we obtain

$$\frac{\partial v_y}{\partial y} = \left[\frac{B_y^2(\epsilon, y) - B^2(x, y)}{8\pi L^2\rho}\right]^{1/2} \quad \text{.....(14)}$$

We would like to demonstrate in our final calculations to what extent compressibility may enhance the merging of the two fields. The thinness of the transition layer ($l \ll L, b$) in which the fluid is expelled suggests that it is not unreasonable to suppose that the compression may sometimes be essentially isothermal; we write

$$\rho = \rho_0(p/p_0)$$

The equation of continuity under steady-state conditions becomes

$$\frac{\partial}{\partial x}(v_x p) = -p\frac{\partial v_y}{\partial y} - v_y\frac{\partial p}{\partial y}$$

At $y = 0$, $\partial v_y/\partial y$ is finite and v_y vanishes. Hence, for $y^2 < b^2$, we write

$$\frac{\partial}{\partial x}(v_x p) \cong -p\frac{\partial v_y}{\partial y} \qquad (15)$$

The hydromagnetic equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{c^2}{4\pi\sigma}\nabla^2\mathbf{B}$$

may be written

$$\nabla \times [\mathbf{v} \times \mathbf{B} - (c^2/4\pi\sigma)\nabla \times \mathbf{B}] = 0$$

when $\partial/\partial t = 0$. Integrating, we find that

$$\mathbf{v} \times \mathbf{B} = (c^2/4\pi\sigma)\nabla \times \mathbf{B} + \nabla\Phi \qquad (16)$$

where Φ is some scalar function of position. Since B_x and $\partial B_x/\partial y$ vanish at $y = 0$, it follows that (16) reduces to

$$v_x B_y = \frac{c^2}{4\pi\sigma}\frac{\partial B_y}{\partial x} + \nabla\Phi$$

in the region about $y = 0$. Since $v_x = 0$ at $x = 0$, we may evaluate $\nabla\Phi$ to be equal to $\partial B_y/\partial x$ at $x = 0$, or

$$v_x B_y = \frac{c^2}{4\pi\sigma}\left[\frac{\partial B_y}{\partial x} - \left(\frac{\partial B_y}{\partial x}\right)_0\right] \qquad (17)$$

To determine the velocity with which the two oppositely directed fields are carried into each other, we must compute v_x outside the transition layer. Outside the transition layer, $\partial B_y/\partial x \cong B/L$ and is negligible. Hence, from (17), we find that the velocity v_∞ at which the fields merge is just

$$v_\infty = \frac{c^2}{4\pi\sigma B_y(\epsilon, y)}\left(\frac{\partial B_y}{\partial x}\right)_0 \qquad (18)$$

We now use (14) to eliminate $\partial v_y/\partial y$, and (17) to eliminate v_x, from (15). We use (13) to eliminate p. Since we restrict ourselves to the region near $y = 0$, we may write $B = B_y$. We obtain

$$\left.\begin{aligned}\frac{\partial}{\partial x}\left\{\left[\frac{\beta^2(y) - B^2(x, y)}{B(x, y)}\right]\left[\frac{\partial B(x, y)}{\partial x} - \left(\frac{\partial B}{\partial x}\right)_0\right]\right. \\ = -\eta\beta(y)[\beta^2(y) - B^2(x, y)]^{1/2}[B_\infty^2(y) - B^2(x, y)]^{1/2}\end{aligned}\right\} \qquad (19)$$

where

$$B_\infty(y) \equiv B_y(\epsilon, y) \qquad (20)$$

$$\beta^2(y) \equiv 8\pi p_0 + B_y^2(\epsilon, y) \qquad (21)$$

$$\eta \equiv 4\pi\sigma p_0^{1/2}/c^2\rho_0^{1/2}L\beta(y) \qquad (22)$$

For convenience, we let

$$\phi \equiv B(x, y)/B_y(\epsilon, y) \qquad (23)$$

$$f(\phi) = (\partial B/\partial x)/(\partial B/\partial x)_0 \qquad (24)$$

$$\xi \equiv B_y(\epsilon, y)/\beta(y) \qquad (25)$$

treating ϕ as the independent variable in place of x. Then (19) may be rewritten

$$f\frac{df}{d\phi} - \frac{f(f-1)}{\phi}\left(\frac{1+\xi^2\phi^2}{1-\xi^2\phi^2}\right) + \frac{\Lambda}{\lambda}\frac{\phi(1-\phi^2)^{1/2}}{(1-\xi^2\phi^2)^{1/2}} = 0 \qquad (26)$$

where the lengths Λ and λ are equal to $B_\infty/(\partial B/\partial y)_0$ and $(\partial B/\partial y)_0/\eta B_\infty^2$, respectively. We note from (23) that $\phi \leq 1$, since $B_y(\epsilon, y)$ is the value to which $B(x, y)$ rises as we leave the transition region, in the vicinity of $x = 0$. ξ is also less than or equal to unity.

We may expand $f(\phi)$ about $\phi = 0$ in ascending powers of ϕ. We obtain

$$\left.\begin{aligned} f(\phi) = 1 - (\Lambda/\lambda)\phi^2 + \tfrac{1}{6}(\Lambda/\lambda)(1 - 5\xi^2 - 2\Lambda/\lambda)\phi^4 \\ + \tfrac{1}{4}(\Lambda/\lambda)[1 + 14\xi^2/3 - 97\xi^4/3 + (16/3)(\Lambda/\lambda)(1 - 3\xi^2) \\ - (32/3)(\Lambda/\lambda)^2]\phi^6 + \cdots \end{aligned}\right\} \quad \ldots(27)$$

We cannot expand $f(\phi)$ in an ordinary power series about $\phi = 1$ $(B = B_\infty)$ because it is not a regular point of the equation. We shall find, however, that when ϕ is close to unity, $f(f-1)/\phi$ is small in comparison to $f\,df/d\phi$. Thus, we may solve the equation by reiteration. The zero-order function $f^{(0)}(\phi)$ satisfies

$$f^{(0)}\frac{df^{(0)}}{d\phi} + \frac{\Lambda}{\lambda}\frac{\phi(1-\phi^2)^{1/2}}{(1-\xi^2\phi^2)^{1/2}} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(28)$$

We shall suppose that compressibility effects are small, $\xi^2 \ll 1$. Then, neglecting terms $O^4(\xi)$, we readily find that

$$f^{(0)}(\phi) = \left(\frac{2\Lambda}{3\lambda}\right)^{1/2}(1-\phi^2)^{3/4}\left\{1 + \frac{\xi^2}{20}(2 + 3\phi^2)\right\} + O^4(\xi) \quad \ldots\ldots(29)$$

Because $\partial B/\partial y$ essentially vanishes once we have left the transition region, we have the boundary condition $f(1) = 0$.

We now return to (26), using $f^{(0)}(\phi)$ in the term involving $f(f-1)/\phi$ and computing $f^{(1)}(0)$ from

$$f^{(1)}\frac{df^{(1)}}{d\phi} - \frac{f^{(0)}(f^{(0)}-1)}{\phi}\left(\frac{1+\xi^2\phi^2}{1-\xi^2\phi^2}\right) + \frac{\Lambda}{\lambda}\frac{\phi(1-\phi^2)^{1/2}}{(1-\xi^2\phi^2)^{1/2}} = 0 \ldots\ldots(30)$$

Noting that

$$\int du(1-u)^{3/4}/u = 2\left\{\frac{2}{3}(1-u)^{3/4} + \arctan(1-u)^{1/4} - \operatorname{arctanh}(1-u)^{1/4}\right\}$$

we readily find that

$$\left.\begin{aligned} f^{(1)}(\phi) = \Bigg\{&\Bigg\{\left(\frac{2\Lambda}{3\lambda}\right)\left[\frac{5}{3}(1-\phi^2)^{3/2} + 2(1-\phi^2)^{1/2} - 2\operatorname{arctanh}(1-\phi^2)^{1/2}\right] \\ &- 2\left(\frac{2\Lambda}{3\lambda}\right)^{1/2}\left[\frac{2}{3}(1-\phi^2)^{3/4} + \arctan(1-\phi^2)^{1/4} - \operatorname{arctanh}(1-\phi^2)^{1/4}\right]\Bigg\} \\ &+ \xi^2\Bigg\{\left(\frac{2\Lambda}{3\lambda}\right)\Bigg[2(1-\phi^2)^{1/2} + \frac{19}{6}(1-\phi^2)^{3/2} - \frac{61}{10}(1-\phi^2)^{5/2} \\ &- 2\operatorname{arctanh}(1-\phi^2)^{1/2}\Bigg] - \left(\frac{2\Lambda}{3\lambda}\right)^{1/2}\Bigg[\frac{2}{3}(1-\phi^2)^{3/4} - \frac{43}{7}(1-\phi^2)^{7/4} \\ &+ \arctan(1-\phi^2)^{1/4} - \operatorname{arctanh}(1-\phi^2)^{1/4}\Bigg]\Bigg\}\Bigg\} \end{aligned}\right\} \quad (30)$$

We note that $f^{(1)}(1) = 0$, as required.

We must now adjust Λ/λ in such a way that the series expansion about $\phi = 0$, given in (27), connects into the reiteration solution about $\phi = 1$, given by (30). We shall require that they meet at $\phi = 3/4$, which we shall find gives a smooth curve; both $f(\phi)$ and $df/d\phi$ appear to be continuous across $\phi = 0.75$. Equating $f(0.75)$ to $f^{(1)}(0.75)$ leads to a transcendental equation, requiring numerical solution. For the case of incompressibility, $\xi = 0$, we find that $\Lambda/\lambda = 0.820$; when $\xi = 0.316$, we find that $\Lambda/\lambda = 0.772$. $f(\phi)$ is shown as a function of ϕ in Figure 3.

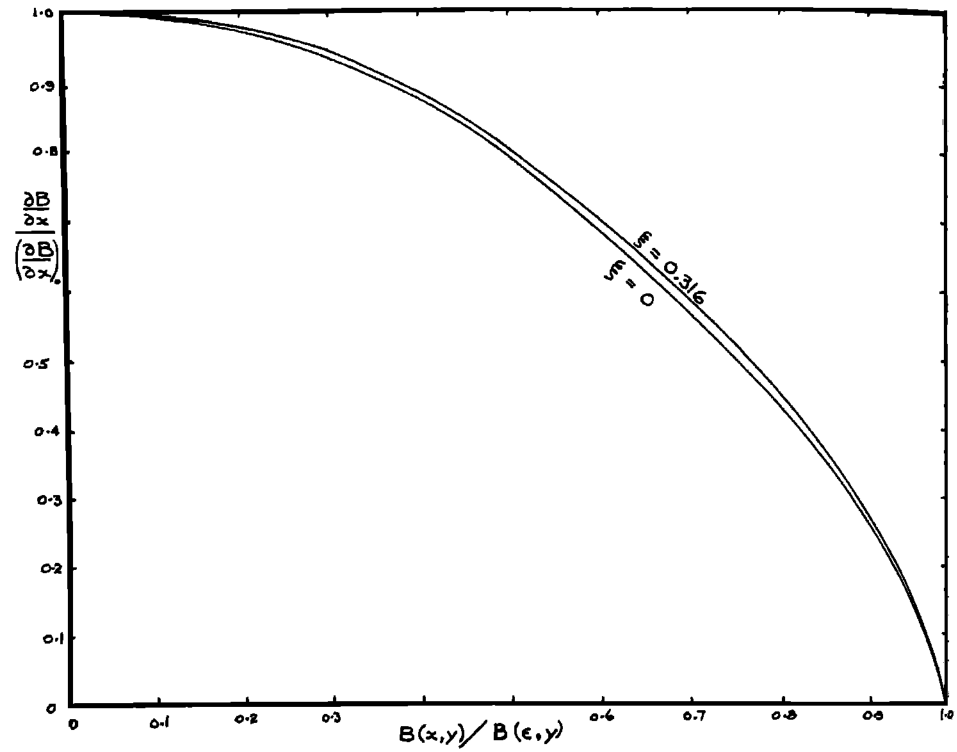

FIG. 3—$\partial B/\partial x$ (in units of $\partial B/\partial x$ at $x = 0$) as a function of $B(x, y)$ (in units of the field density outside the transition region) for incompressible ($\xi = 0$) fluid motion and for slightly compressible ($\xi = 0.316$) fluid motion

From (20), (21), (22), and (25), it is readily shown that

$$\frac{1}{B_y(\epsilon, y)}\left(\frac{\partial B_y}{\partial x}\right)_0 = \left[\left(\frac{4\pi\sigma\xi}{c^2L}\right)\left(\frac{p_0}{\rho_0}\right)^{1/2}\left(\frac{\lambda}{\Lambda}\right)\right]^{1/2}$$

(18) becomes

$$v_\infty = c\,\frac{\xi^{1/2}}{2\sqrt{\pi}}\left(\frac{\lambda}{\Lambda}\right)^{1/2}\left[\frac{(p_0/\rho_0)^{1/2}}{L\sigma}\right]^{1/2}$$

$$= \frac{c}{2^{5/4}\sqrt{\pi}}\left(\frac{\lambda}{\Lambda}\right)^{1/2}\left(\frac{C}{\sigma L}\right)^{1/2}(1 - \xi^2)^{1/2}$$

C is the hydromagnetic velocity $B_y(\epsilon, y)/(4\pi\rho_0)^{1/2}$ outside the transition region

Hence, when $\xi = 0$, we have

$$v_\infty = 0.263\, c(C/\sigma L)^{1/2} \qquad (32)$$

when $\xi = 0.316$,

$$v_\infty = 0.253\, c(C/\sigma L)^{1/2} \qquad (33)$$

(32) and (33) serve as crude estimates of the numerical factors omitted in the dimensional arguments presented in Section I. We see that, in terms of C, v_∞ decreases slightly with increasing compressibility: The gradient $(\partial B/\partial y)_0$ across the transition region tends to be enhanced by the compression, as indicated by the smaller value of Λ/λ, but the expulsion of the fluid is slower because of the increased density upon compression; the net effect is a slight decrease in v_∞ when related to the hydromagnetic velocity in the uncompressed gas. The net effect is slight. We must exercise caution, however, in applying our result, that compressibility slightly decreases the rate at which the fields merge, because the sign of the effect depends critically on the form of $\partial v_y/\partial y$ which we have assumed in constructing (19).

IV. CONCLUSION

Sweet's mechanism means that the reconnection of the lines of force of magnetic fields of scale L is not limited to the long time of $L^2\sigma/c^2$ and the slow merging velocity $c(c/\sigma L)$ which one would obtain from the diffusion equation

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{c^2}{4\pi\sigma} \nabla^2 \mathbf{B}$$

When the velocity term $\nabla \times (\mathbf{v} \times \mathbf{B})$ is included, the merging velocity of two fields becomes of the order of $c(C/\sigma L)^{1/2}$, which may be very much larger. C is the hydromagnetic velocity $\mathbf{B}/(4\pi\rho)^{1/2}$. The ratio of Sweet's velocity of merging to the diffusion velocity is $(C/c)^{1/2}\,(\sigma L/c)^{1/2}$, which becomes large for large electrical conductivity, large scale, and/or large hydromagnetic velocity. Two fields need not be antiparallel for the Sweet's mechanism to work effectively: In Figure 4, we show schematically the process of merging two flux tubes which are perpendicular to each other; the process is to be compared with that computed from the diffusion equation alone (Parker and Krook, 1956).

We suggest that Sweet's mechanism may be of great importance in producing reconnection of the lines of force of a magnetic field into a configuration such that its energy becomes available for mechanical motions, etc. A solar flare may be an example of this (Parker, 1957).

We also suggest that the high electrical conductivity (Storey, 1954) of the gas within a few earth's radii of our planet does not necessarily exclude the penetration of exterior magnetic fields; $\sigma = 10^{13}$ esu, $L = 10^9$ cm, and a density of 10^{-21} gm/cm^3 leads to a merging velocity of the order of 0.1 km/sec: We suggest that the arguments we have presented elsewhere (Parker, 1956) against the Chapman-Ferraro ring-current model of the geomagnetic storm may lose some of their force.

Finally, we wonder if it is possible that Sweet's mechanism might modify somewhat the diffusion and dissipation of the magnetic field in hydromagnetic turbulence.

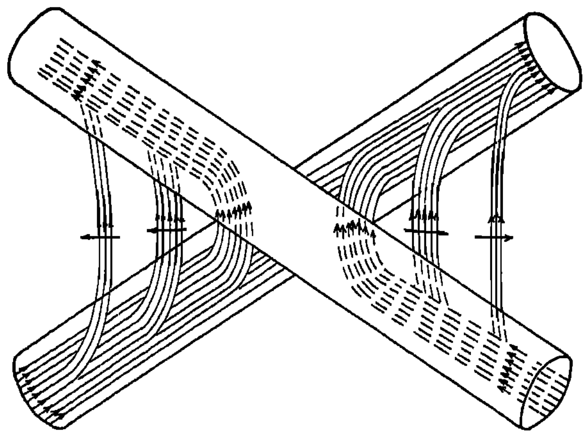

Fig. 4—Schematic drawing of the merging of two perpendicular flux tubes by Sweet's mechanism. The fluid squeezes out of the region of contact of the two tubes by flowing along the lines of force into the arms of the tubes. Following severing and reconnection of the lines of force, the tension in the reconnected lines tends to make them pull away from the region of contact and follow a shorter path between the tubes, as shown.

References

Sweet, P. A. (1956); Proceedings of the International Astronomical Union Symposium on Electromagnetic Phenomena in Cosmical Physics, Stockholm, 1956.

Parker, E. N., and M. Krook (1956); Astroph. J., 124, 214.

Parker, E. N. (1957); Phys. Rev., 107, 830 (1957).

Storey, L. R. O. (1954); Phil. Trans. R. Soc., A, 246, 113.

Parker, E. N. (1956); J. Geophys. Res., 61, 625.