Modeling of ionospheric convection pattern with SuperDARN data using localized vector-valued basis functions

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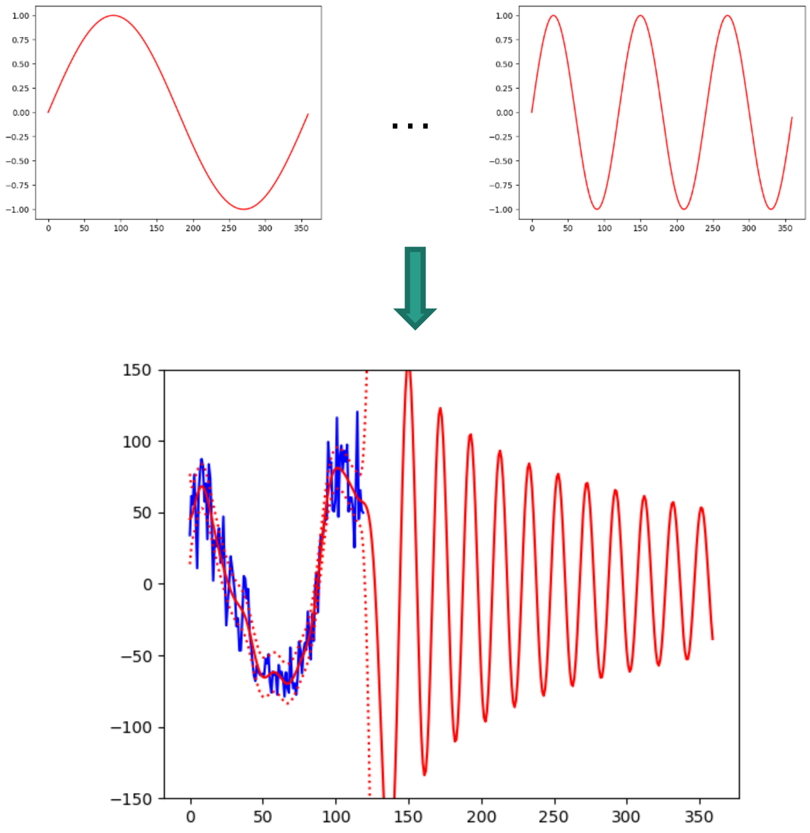
Outline

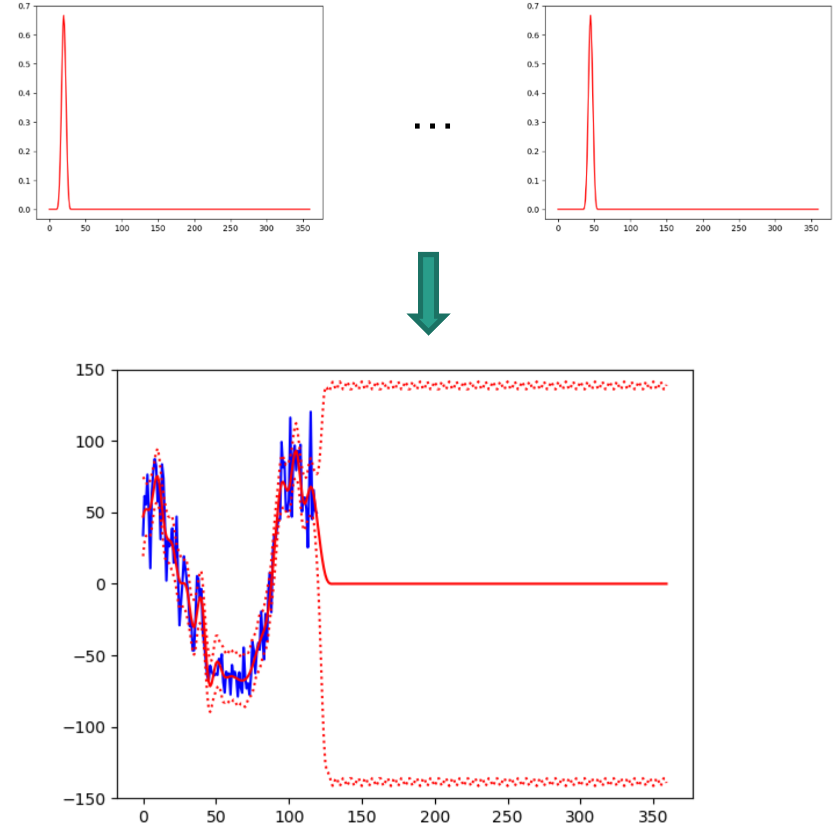
- The Super Dual Auroral Radar Network (SuperDARN) covers wide areas of high-latitude and mid-latitude ionosphere. It provides useful information on global plasma drift velocity distribution.
- However, each radar observes the line-of-sight component of the plasma velocity. There are also some wide gaps in its spatial coverage.
- We assume divergence-free condition to obtain a reasonable estimate even if data are insufficient.
- We use local basis functions for representing the velocity distribution in order to avoid artifacts due to wide data gaps. The use of the local basis functions allows us to fill the data gaps with another empirical model.

Two types of basis functions

1-D case



With global basis functions (Fourier)



With local basis functions (B-spline)

Generalization of SECS

- The ionospheric plasma drift velocity distribution can be assumed to be divergence-free (no source, no sink).
- The divergence-free vector field can be represented by a stream function Ψ as follows:

$$\boldsymbol{V}(\boldsymbol{r}) = -\hat{\boldsymbol{r}} \times \nabla \Psi.$$

- We expand the stream function Ψ by using localized basis functions:

$$\Psi(\boldsymbol{r}) = \sum_i w_i \psi(\boldsymbol{r}, \boldsymbol{r}_i).$$

- Thus,

$$\boldsymbol{V}(\boldsymbol{r}) = -\hat{\boldsymbol{r}} \times \nabla \Psi = -\sum_i w_i \hat{\boldsymbol{r}} \times \nabla \psi(\boldsymbol{r}, \boldsymbol{r}_i).$$

Generalization of SECS

- Defining vector-valued localized basis function:

$$\boldsymbol{v}(\boldsymbol{r},\boldsymbol{r}_i) = -\boldsymbol{e}_r \times \nabla \psi(\boldsymbol{r},\boldsymbol{r}_i),$$

  we obtain

$$\boldsymbol{V}(\boldsymbol{r}) = \sum_i w_i \boldsymbol{v}(\boldsymbol{r},\boldsymbol{r}_i).$$

- If $\psi(\boldsymbol{r},\boldsymbol{r}_i) = 2\log\left|\sin\frac{\Delta\theta'}{2}\right|$ where $\theta' = \arccos\left(\frac{\boldsymbol{r}\cdot\boldsymbol{r}_i}{R^2}\right),$

$$\boldsymbol{v}(\boldsymbol{r},\boldsymbol{r}_i) = \boldsymbol{e}_{\phi,i}\cot\frac{|\Delta\theta'|}{2}.$$

This is the original divergence-free SECS basis function.

Basis based on spherical Gaussian

- We represent the divergence-free velocity field by

$$\boldsymbol{V}(\boldsymbol{r}) = \sum_i w_i \boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}_i), \quad \left(\boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}_i) = -\hat{\boldsymbol{r}} \times \nabla \psi(\boldsymbol{r}, \boldsymbol{r}_i)\right).$$

- We choose the spherical Gaussian function for ψ:

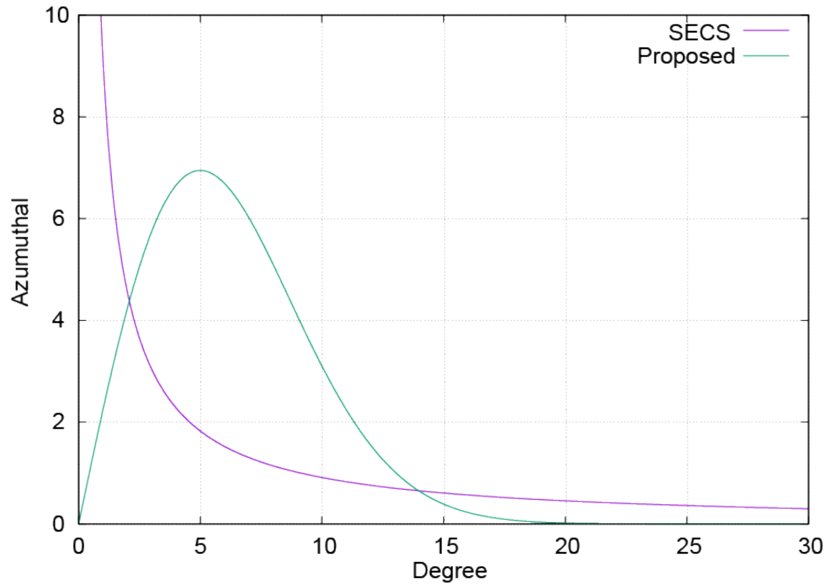
$$\psi(\boldsymbol{r}, \boldsymbol{r}_i) = \exp\left[\eta\left(\frac{\boldsymbol{r} \cdot \boldsymbol{r}_i}{R_I^2} - 1\right)\right] = \exp\left[\eta\left(\cos\theta' - 1\right)\right],$$

and obtain the following divergence-free basis function

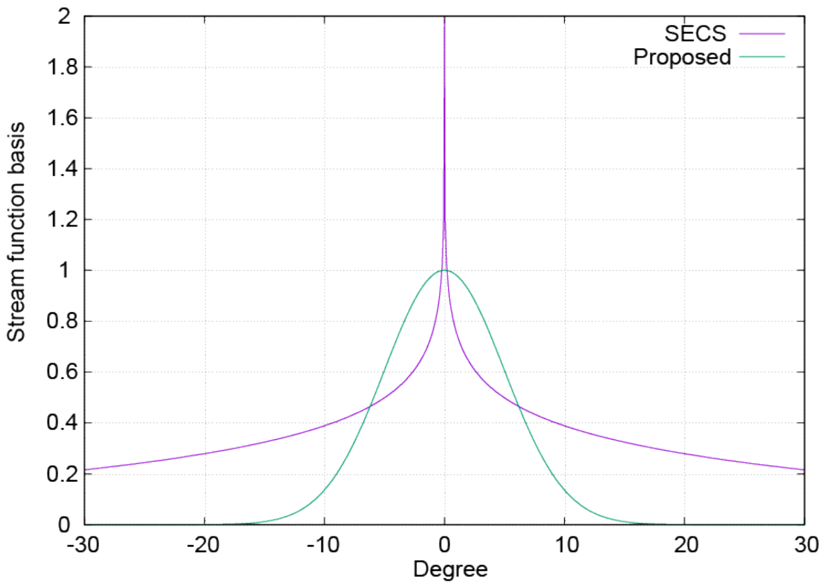
$$\boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}_i) = (\eta \boldsymbol{r}_i \times \boldsymbol{r}) \exp\left[\eta\left(\frac{\boldsymbol{r} \cdot \boldsymbol{r}_i}{R_I^2} - 1\right)\right].$$

Shape of the functions

$$\boldsymbol{v}(\boldsymbol{r},\boldsymbol{r}_i) = -\boldsymbol{e}_r \times \nabla \psi(\boldsymbol{r},\boldsymbol{r}_i),$$



$$\psi(\boldsymbol{r},\boldsymbol{r}_i)$$



Distribution of node points

- When we use the expansion of our basis functions

$$\boldsymbol{V}(\boldsymbol{r}) = \sum_i w_i \boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}_i),$$

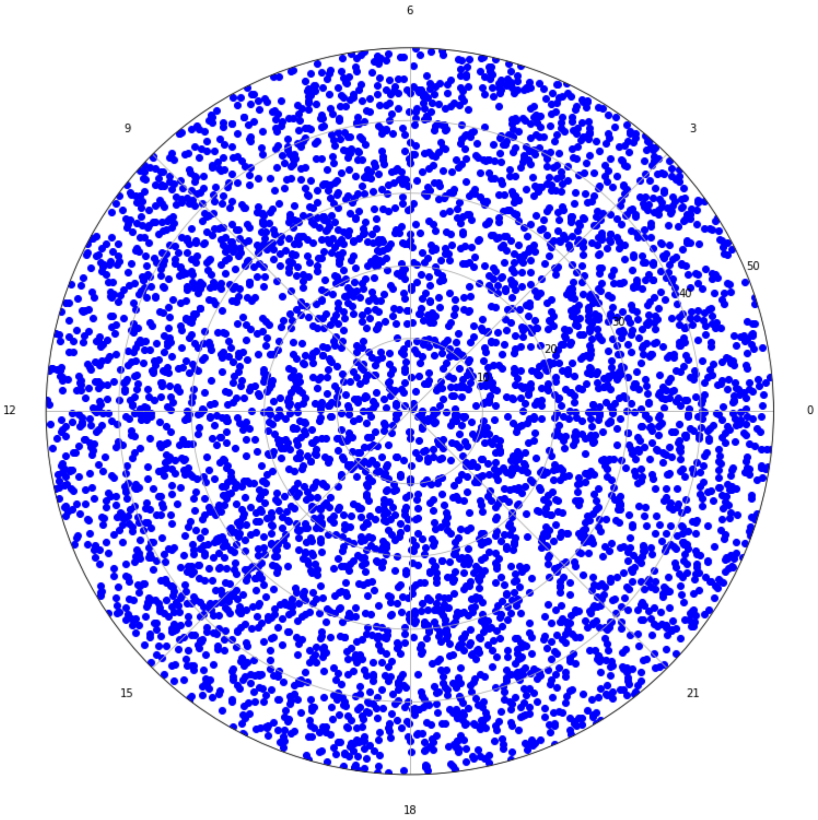
the node points $\boldsymbol{r}_i$ can be placed arbitrarily.

- We placed 2500 node points randomly and uniformly distributed in the region above 40 degree in latitude.
- This corresponds to the Monte Carlo convolution:

$$\boldsymbol{V}(\boldsymbol{r}) = \int_S w(\boldsymbol{r})\boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}')d\boldsymbol{r} = \int_0^{\pi/2}\int_0^{2\pi} w(r)\boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}')R^2 \sin\theta d\theta d\phi.$$

Distribution of node points

- We placed 2500 node points randomly and uniformly distributed in the region above 40 degree in latitude.



Use of empirical model

- An empirical model is also referred to in estimating the velocity distribution.
- The data gaps are filled with the empirical model.
- We assume the weight $\boldsymbol{w}$ can be decomposed into the model-based value $\boldsymbol{\zeta}$ and the residual $\boldsymbol{\beta}$:

$$\boldsymbol{w} = \boldsymbol{\zeta} + \boldsymbol{\beta}.$$

- The model-based value is determined so as to fit an empirical model by Weimber 2001.
- The residual $\boldsymbol{\beta}$ is estimated with the Kalman filter.

Kalman filter

- We assume the temporal evolution of the weights $\boldsymbol{w}$

$$p(\boldsymbol{\beta}_k \mid \boldsymbol{\beta}_{k-1}) = \mathcal{N}(\alpha \boldsymbol{\beta}_{k-1}, \mathrm{Q}).$$

- The residual component can then be estimated with the following Kalman filter algorithm:

Prediction:

$$\boldsymbol{\beta}_{k|k-1} = \alpha \boldsymbol{\beta}_{k-1|k-1},$$

$$\mathrm{P}_{k|k-1} = \alpha^2 \mathrm{P}_{k-1|k-1} + \mathrm{Q}.$$

Filtering:

$$\boldsymbol{\beta}_{k|k} = \boldsymbol{\beta}_{k|k-1} + (\mathrm{P}_{k|k-1}^{-1} + \mathrm{H}_k^T \mathrm{R}_k^{-1} \mathrm{H}_k)^{-1} \mathrm{H}_k^T \mathrm{R}_k^{-1} (\boldsymbol{y}_k - \mathrm{H}_k \boldsymbol{\beta}_k),$$

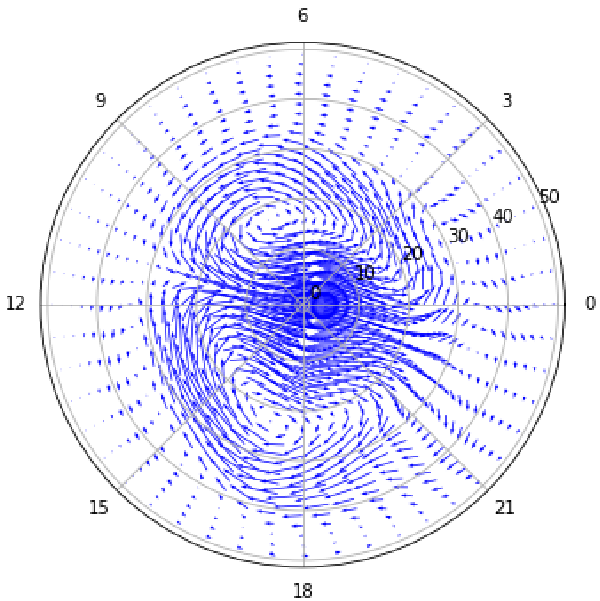
$$\mathrm{P}_{k|k} = (\mathrm{P}_{k|k-1}^{-1} + \mathrm{H}_k^T \mathrm{R}_k^{-1} \mathrm{H}_k)^{-1}.$$

Estimation

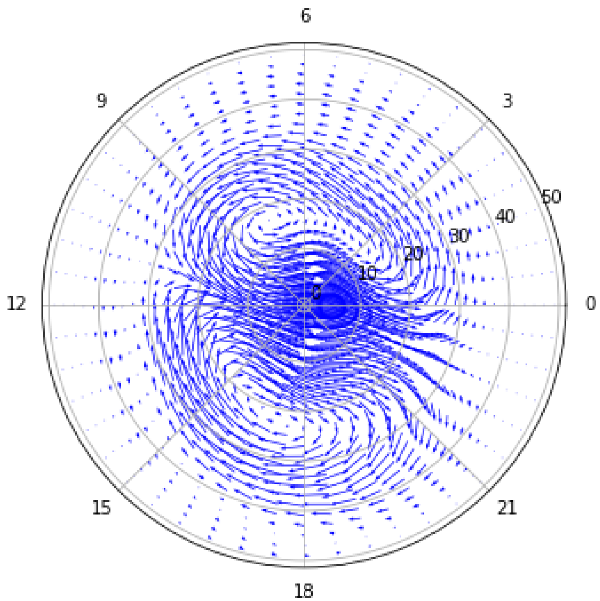
- The covariance matrix Q_k considers spatial correlation.
- In the matrix R_k, we consider only the correlation between the records in the same beam direction (different range gate).
- The parameters are determined by maximizing the marginal likelihood:

$$p(\boldsymbol{y}_{1:K} \mid \boldsymbol{\theta}) = \prod_k \int p(\boldsymbol{y}_k \mid \boldsymbol{w}_k)\; p(\boldsymbol{w}_k \mid \boldsymbol{\theta})\; d\boldsymbol{w}_k .$$

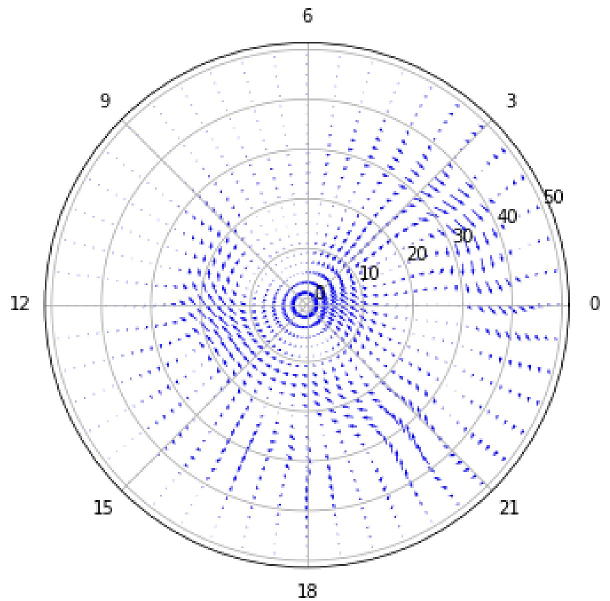
Result



Estimated velocity distribution



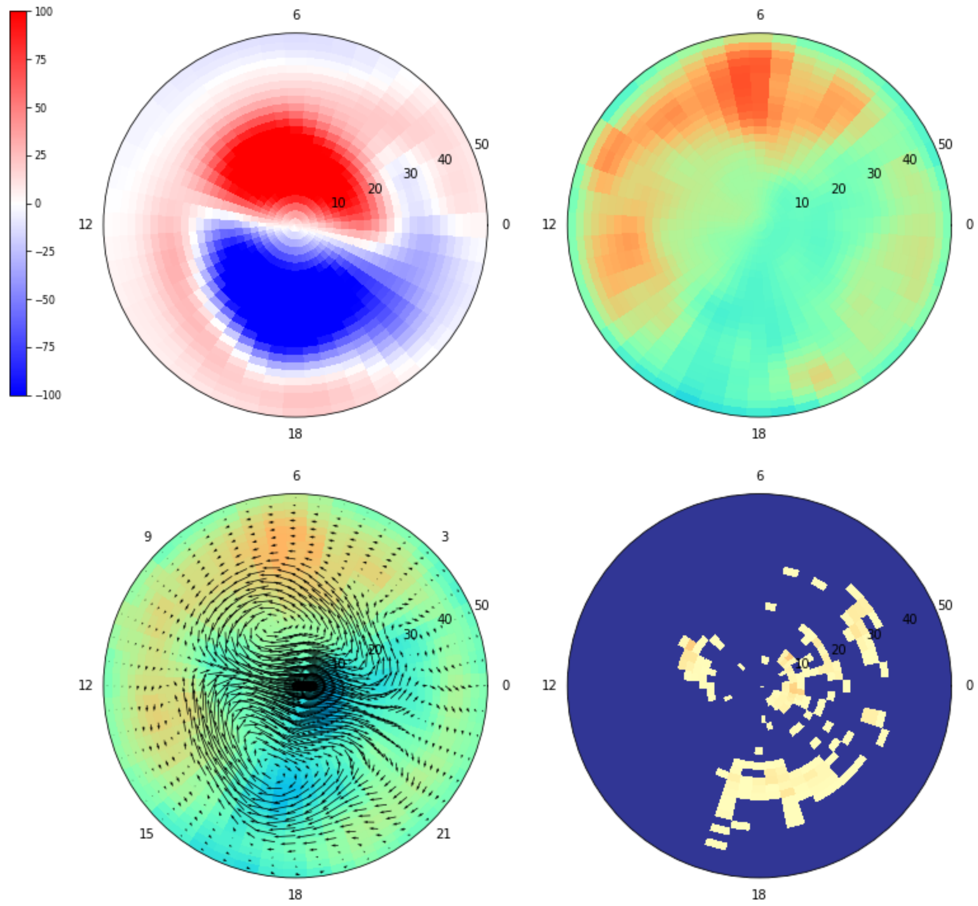
Original velocity distribution



Difference

0800 UT, Mar. 17, 2015

Result



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Summary

- We have proposed a framework for obtaining global flow vector distribution on a sphere.
- In our framework, the vector field is represented by Monte Carlo convolution of localized basis functions. The basis function is derived from a spherical Gaussian function and satisfy divergence-free conditions.
- The use of the localized basis functions allows us to fill the data gaps of the radar data coverage with another model of the velocity distribution.
- Temporal evolution is also considered by using Kalman filter in order to overcome the data missing.